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Adaptive Variational Sinogram Interpolation of Sparsely Sampled CT Data

1 Introduction

Tomographic image reconstruction is applied in various fields of medical imaging. Conventional CT scanners and C-arm systems e.g. allow a tomographic reconstruction of a 2D or 3D image of a patient. Therefore a rotating X-ray device is used to acquire a sequence of angiographic X-ray images. Unfortunately in some clinical applications like limited angle tomography or cardiac CT the sinogram data is sparse or in the sense of the reconstruction theory underlying fourier-slice-theorem partially incomplete.

1.1 Problem Statement

Sparse and/or incomplete sinogram data can lead to various kinds of reconstruction artifacts. Therefore sinogram interpolation of missing data is desirable.

1.2 Goals for Sinogram Interpolation

- Preserve edges along a detector line.
- Smooth sinogram data along sinuodal traces.
- Streak artifact reduction in the reconstructed image while preserving object structure.

1.3 Breakthrough Work Presented

We present a new variational approach for sinogram data interpolation prior to reconstruction based on the work of Weickert [7] and compare the reconstructed images using the interpolated sinogram with a standard interpolation technique of *spectral deconvolution*.

1.4 Spectral Deconvolution

- observed sinogram $g(n)$ (of size $N \times \Phi$) - where N is the number of detector pixel and Φ the scan angle - and a gap image mask $w(n)$ that has zero lines at projection angles ϕ where no data is given otherwise it is one.
- $f(n)$ is the ideal sinogram that we want to find.
- Formulation of incomplete sinogram image as: $g(n) = f(n)w(n)$,
- Solve iteratively for $f(n)$ using spectral deconvolution as introduced by Til Aach [1],[2].

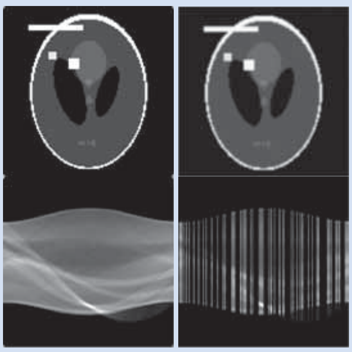

Figure 1: Modified Shepp-Logan head phantom of size 128^2 (TL), its sinogram (BL), reconstructed image using full sinogram data (TR) and sinogram with gaps (BR), where 66.2% of the points were missing.

2 Variational Interpolation Approach

For a CT slice reconstruction we have a sinogram with missing samples along lines (gaps) where the gap positions are exactly known. Our key idea comes from image inpainting (cf. e.g. [4]) where several key frames of an image sequence are given and new frames are interpolated in-between.

- the ideal and the incomplete sinograms are considered as functions $f, g : \Omega \to \mathbf{R}$ in the sinogram domain $\Omega \subset \mathbf{R}^d$
- f is obtained by minimizing the energy functional

$$E(f) = \int_\Omega w(x)(g-f)^2 + (1-w(x))\Psi(s^2)\,dx \ ,$$

  where $x \in \Omega$, $s^2 = |\nabla^k f|^2$ and $k \in \{1, 2\}$. The first term ensures the equivalence of g and f at positions, where the sinogram data is known, and $\Psi : \mathbf{R} \to \mathbf{R}$ is a regularizing term filling in the missing information.
- This leads to the Euler-Lagrange equations

$$\begin{aligned} -L(f) &= 0 \text{ if } w(x) = 0 \\ f &= g \text{ if } w(x) = 1 \quad , \\ \tfrac{\partial f}{\partial n} &= 0 \text{ on } \partial\Omega \end{aligned}$$

  with $x \in \Omega$ and an elliptic differential operator L.
- System of PDEs is discretized by finite differences in space and for the nonlinear variants by an explicit Euler forward scheme in time. Afterwards it is solved by either a simple Gauss Seidel iteration or if necessary (to enable a fast transport of the information through large gaps) by a multigrid solver (cf. [6]).

We have implemented various linear regularizers with $\Psi(s^2) = s^2$ such as

1. isotropic harmonic diffusion (IH)

$$L(f) = \Delta f$$

   (corresponds to linear interpolation in 1D)
2. biharmonic diffusion (BI)

$$L(f) = \Delta^2 f$$

   (corresponds to cubic spline interpolation in 1D, in higher dimensions to using radial basis functions [3]).
3. anisotropic harmonic diffusion (AH)

$$L(f) = \nabla \cdot (D \nabla f)$$

   The idea for here is to smooth mainly in the direction of the optical flow (cf. [5]). Therefore the tensor

$$D = \begin{pmatrix} (u/v)^2 + \alpha_1 & u \\ u & (1-(u/v)^2) + \alpha_2 \end{pmatrix}$$

   with $\alpha_1, \alpha_2 > 0$ was constructed using motion information from the left to the right side of a gap approximated by the normalized optical flow vector $(u/v, 1)^T$. v is the size of a gap measured in number of pixels. For 1D optical flow we approximate the spatial resp. temporal derivative g_x and g_t of g by finite differences, where g_t is computed between the two sides of a gap. We have

$$\begin{aligned} u &= 0 \quad \text{ if } g_x = 0 \\ u &= -\tfrac{g_t}{g_x} \text{ if } g_x \neq 0 \end{aligned} \ .$$

   In higher dimensions we get the optical flow vector by solving a system of PDEs using multigrid.
4. isotropic nonlinear diffusion (IN)

$$L(f) = \nabla \cdot (\Psi'(s^2) \nabla f)$$

   with a Charbonnier diffusivity $\Psi'(s^2) = \frac{1}{1+s^2/\lambda^2}$ and a contrast parameter $\lambda > 0$.

3 Results

For computing the sinogram and the reconstruction we used Matlab in 2D and a C++ implementation in 3D. We investigated a scenario of a fan-beam short-scan and introduced artificially short sequences of missing sinogram data as shown in Fig. 1.

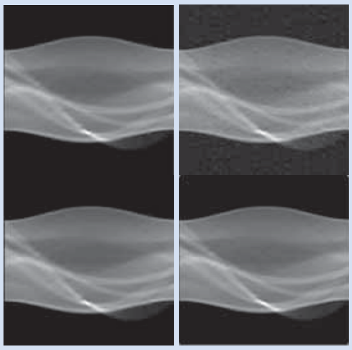

Figure 2: Interpolation of incomplete sinogram data using isotropic diffusion (TL), spectral deconvolution (TR), anisotropic diffusion (BL) and biharmonic diffusion (BR). One can observe that edges along a detector line is preserved while smoothing along the sinuodal traces. The corresponding reconstructions are shown in fig. 3 and 4.

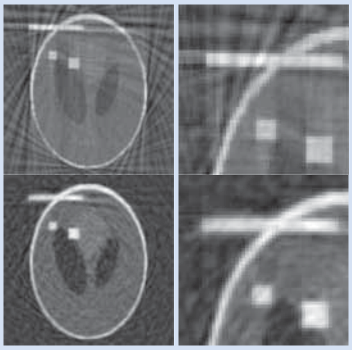

Figure 3: Reconstructed images without interpolation (top) and using spectral deconvolution for sinogram data interpolation of the missing data (bottom).

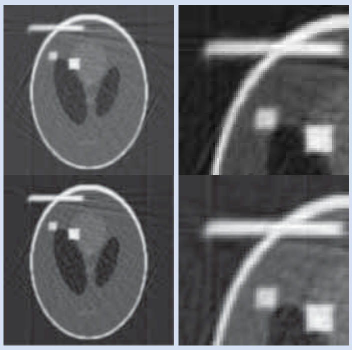

Figure 4: Reconstructed images using a partially interpolated sinogram. The missing sinogram data is interpolated using anisotropic diffusion regularization (top) and biharmonic regularization (bottom).

Table 1: L_2-norm of the reconstruction error E_r for different interpolation methods (gapped sinogram without interpolation gave 16.0). All error values have to be scaled by 10^{-4} anf the number of Gauss Seidel iterations needed is given. Times measured on a Pentium M 1400 MHz Laptop.

Method	# Iter.	E_r	α	Time (sec.)
IH	200	9.60	(0.01,0.99)	0.67
BI	1000	9.54	(0.1,0.9)	6.48
IN	300	9.58	(0.1,0.9)	61.39
AH	300	9.52	(0.1,0.9)	2.23
SD	1000	9.93	-	21.02

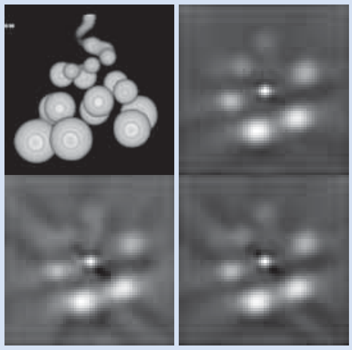

Figure 5: 3D phantom (TL) of size 64^3 and center slices of reconstructed volumes using full sinogram (TR), sinogram with gaps (BL) and sinogram interpolated with BI interpolation (BR). 60% of the sinogram data ($N = 256 \times 128$, $\Phi = 225$) was missing.

4 Summary and Conclusions

- We present various kinds of variational PDE based methods to interpolate missing sinogram data for tomographic image reconstruction
- Using the observed sinogram data we inpaint the projection data by diffusion
- To overcome the problem of contour blurring during sinogram interpolation we consider nonlinear and anisotropic diffusion based regularizers and include optical flow information in order to preserve the sinuodal traces corresponding to object contours in the reconstructed image without introducing new kind of artifacts.
- We compare our results to a spectral deconvolution based interpolation and show that the method can easily be extended to 3D

References

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