Singular Current Sources in PDEs

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Abstract:

- Reconstruction of electrical behaviour inside a human head
- Data source: non invasive measurements like EEG or MEG
- Important in neurology and neurosurgery (e.g. epilepsy)
- From Maxwell equations for electrodynamics derive a PDE

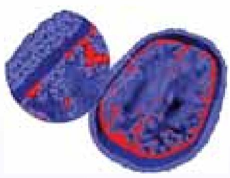
$$\nabla \cdot (\sigma \nabla \Phi) = \nabla \cdot \vec{j}_s \quad \text{in } \Omega$$

$$< \sigma \nabla \Phi, \vec{n} > = g \quad \text{on } \partial\Omega$$

with a dipolar current source

$$\vec{j}_s = \vec{M}\delta(\vec{x} - \vec{x}_0)$$

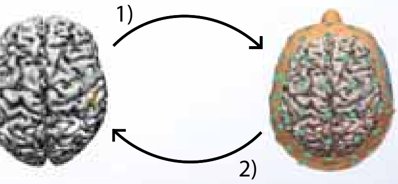
- Distinguish between different compartments with different conductivities



Parts of Source Reconstruction:

1) **Forward Problem**: Calculate the potential at EEG electrodes caused by a given source configuration

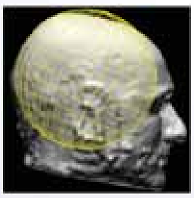
2) **Inverse Problem**: Determine source from a given EEG potential

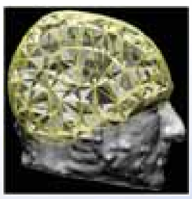


Goal: Define a suitable dipole model to treat the singular Dirac distribution on the right hand side of the PDE

Finite-Element Approach:

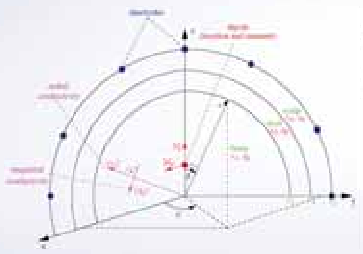
- Discretization using a Finite-Element approach
- For validation discretize a spherical approximation of the human brain
- After validation use realistic FE models of the brain



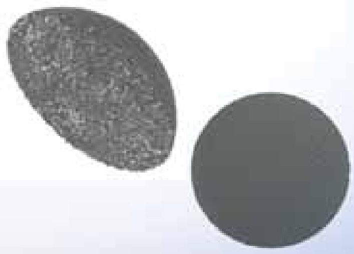


Discretized Models:

3-layer model with different conductivity in each layer



Tetrahedral and hexahedral models for FE approach



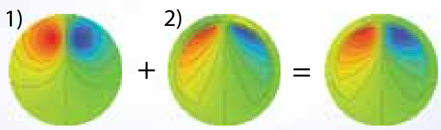
Dipole Models

Blurred Dipole Model

- Approximation of the dipole moment by a whole collection of closely neighbored sources and sinks
- Well-known in mechanical engineering: small forces in combination with long lever arms have the same effect as large forces in combination with short lever arms
- Goal: spread singularity in strength and position
- Good results in realistic head models
- Problems with anisotropy in source element

Subtraction Dipole Model

- Split potential and conductivity
  1) Singularity potential: contains the singularity, can be computed analytically
  2) Correction potential: leads to a poisson equation with inhomogeneous von Neumann boundaries



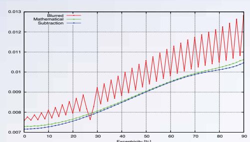
- Accurate results
- Sources in anisotropic medium possible
- High influence of coefficient jumps

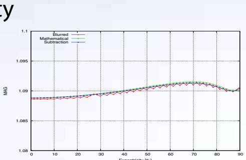
Mathematical Dipole Model

- Apply Green's formula to the RHS
- Use the definition of the Dirac distribution and its integration
- Calculate node entries of the source element
- Leads to a sparse matrix
- Accurate results
- Problems with stability near the singularity
- Extension to a higher order approximation (e.g. Zenger Correction) possible

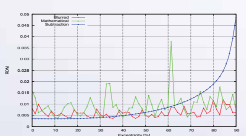
Results and Conclusion

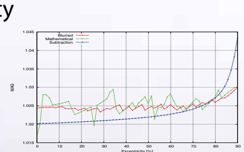
- Relative Difference Measure and Magnitude Error on a 2 mm hexahedron mesh. Tangential dipol with different excentricity





- Relative Difference Measure and Magnitude Error on a 2.5 mm tetrahedron mesh. Tangential dipole with different excentricity





- All models show good results
- Small meshsize leads to smaller error
- Mathematical and Blurred Model depend on distance to the next node
- Problems with conductivity jumps
- Radial sources cause higher error than tangential sources