Accuracy aspects for finite element method based source analysis

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Introduction

Two important aspects in Finite Element (FE) method based volume conductor modeling in EEG and MEG source analysis are

a) **the way of modeling the current dipole**

b) **the choice of the elements and the quality of the corresponding mesh**.

We developed and implemented three different numerical ways of FE-based current dipole modeling

a1) a **subtraction** potential approach [1,2,8,10,14,16]

a2) a direct potential approach using **Green**'s method [9,11]

a3) a direct potential approach using Saint **Venant**'s principle [3,13,14]

and mesh generation algorithms and FE Ansatz-functions for the following three element types

b1) surface-based tetrahedra (**tet**) [2,8,12,13,14,15]

b2) voxel-based regular hexahedra (**hex**) [10, 13]

b3) voxel-based node-shifted (surface-smoothed) hexahedra (**ns-hex**) [4].

In this study we first evaluate the interplay between the methods from a) with those from b) with regard to their accuracy in a multi-layer sphere model [5]. We finally compute and visualize the potential distributions of the subtraction approach in a geometry-adapted ns-hex mesh.

Theory

a) FE current dipole modeling

a1) The ***subtraction*** potential approach divides the total potential into a singularity potential (dipole in infinite region of homogeneous conductivity) and a correction potential. When subtracting the differential equation for the singularity potential from the starting potential equation, a singularity free Poisson-problem with inhomogeneous Neumann boundary conditions for the correction potential results. Our 3D FE approach for anisotropic head models is closely related to the 2D implementation in [1]. We additionally give a numerical analysis with a correction potential existence and uniqueness proof and FE convergence properties [16].

a2) Green's formula can be used for the right-hand-side (RHS) of the starting Poisson-like potential equation in the variational FE formulation. The RHS is then identical to an evaluation of the scalar product of the dipole moment with the gradient of the Ansatz-function evaluated at the source position. For linear Ansatz-functions, the gradient is constant and non-zero only over the source element, so that the resulting FE linear equation system has only 4 (tet) or 8 (hex) non-zero RHS entries (identical to monopolar loads). This approach was used, e.g., in [9,11].

a3) Saint ***Venant***'s principle states that the specific (fine) details of load application do not influence the results observed in some distance away from the locus of load application. Following [3], a dipole can be modeled by placing monopolar sources on all neighboring FE nodes to that FE node which is closest to the source. By means of solving a local Tikhonov-Phillips regularization problem, the monopolar loads are computed so that, multiplied with their "lever arms" (distance of the node to the source), the dipole moment is optimally matched.

b) FE mesh generation

b1) Tetrahedra FE meshing uses pre-segmented triangulated tissue surfaces and a Delaunay-criterion for the generation of compact and regular tetrahedra [2,3,12,13,15].

b2) Hex: Voxels from a segmented MR volume can be used directly as hexahedral elements, possibly reducing resolution by prior subsampling of the volume [4,10,13].

b3) Ns-hex: In order to increase conformance to the real geometry and to mitigate the stair-case effects of a voxel mesh, it was proposed to shift hexahedra nodes on material interfaces to obtain smoother and more accurate boundaries [4]. The applied node-shift factor **ns** (see Fig.1) has to ensure that interior angles at element vertices remain convex and Jacobian determinants remain positive.

Methods

The quasi-analytical series expansion formulas [5] were used as a basis for our accuracy studies. Multi-layer sphere models were discretized with the different meshing techniques. Tetrahedra meshing was performed with CURRY (http://www.neuro.com), while we used the software VGRID (http://www.simbio.de) for hexahedra meshing and chose **ns**=0.49 as node-shift factor (Fig.1). For all forward EEG simulations, the software NeuroFEM-COLSAMM (http://www.simbio.de, see [7] for COLSAMM) was used with linear FE Ansatz-functions and an Algebraic MultiGrid preconditioned Conjugate Gradient (AMG-CG) method for solving the resulting FE linear equation systems [12]. If not further indicated, a relative AMG-CG accuracy of 10^{-8} was used. 134 electrodes were distributed in a most regular way over the outer sphere surface. The topography error Relative Difference Measure (RDM) and the MAGnification error (MAG) [2,8,10] between quasi-analytic and numeric results at those measurement sensors were evaluated for dipoles with fixed x and z and varying (in 1mm steps) y-coordinate (depths) and either tangential or radial orientation. The eccentricity was limited to a percentage of the inner layer depending on the number of compartments, because it can be expected that the dipole is at least 2mm below the surface in the middle of the grey matter compartment. Dipole strengths of 1nAm were used. Furthermore, a three tissue realistically-shaped head model was segmented from a T1- and Proton-Density-weighted MR dataset[15]. BioPSE (http://software.sci.utah.edu/biopse.html) was used for potential visualization.

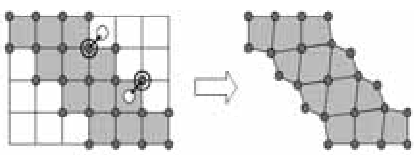

Figure 1

Concept of the hexahedra node-shift in a 2D scenario: On the left, the procedure is illustrated for two boundary nodes, one is moved outside and the other inside towards the centroids of their minority elements. The node-shift factor ns indicates the performed percentage of this move. The final result, a smoothed boundary representation using deformed hexahedra, is shown on the right side.

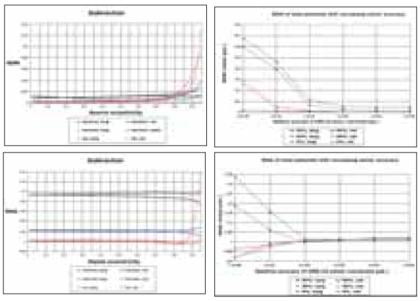

Figure 2

Subtraction approach in three compartment sphere model (90, 80 and 70mm; 0.33, 0.0042 and 0.33S/m) for sources at eccentricities of 0 to 95%. Left column: RDM (top) and MAG (bottom) for a 1mm hex (3,130K nodes, 3,053K elements), a 2mm hex (398K nodes, 378K elements) and a 2mm tet (234K nodes, 1,413K elements) mesh. Right column: RDM (top) and MAG (bottom) (2mm hex) of the total potential with increasing relative AMG-CG solver accuracy for the FE correction potential. Sources at three different depth (0%, 50% and 95% eccentricity) were examined.

Results and discussion

Dipole modeling: As Figs.2 and 3 show, one important advantage of the subtraction approach over the direct methods is that the error curves (and thus the cost functions during inverse optimization) are smooth, while the accuracy of both Venant's and Green's direct potential methods are oscillating. While Venant performs best for sources on FE nodes (i.e. x,y and z are even numbers in Fig.3), Green's method performs best if the source is positioned in the center of an element (x,y and z odd numbers). This can also be observed in tetrahedra models (not shown). On the other side, the subtraction approach with linear Ansatz-functions is computationally more expensive and more sensitive to conductivity jumps in source vicinity if the source is pointing towards the jump (see larger errors of subtraction method for eccentric radial sources in Figs.2 and 3).

For the subtraction approach, highest relative AMG-CG solver accuracies are needed for the most eccentric sources with 10^{-4} being sufficient for the whole eccentricity range (Fig.2, right column).

Element choice and meshing: Figs.2 and 3 show that the 2mm ns-hex model overall performs best with regard to accuracy and computational complexity with the exception of sources within or in the vicinity of a deformed element. For such sources, the node-shift causes numerical problems for both Green and subtraction. We therefore also chose an ns-hex approach for the realistic head model visualizations (Fig.4), while limiting the node-shift to the skin and skull surfaces. 1mm hex modeling avoids complicated mesh generation, but is computationally expensive. The subtraction approach in non-locally refined tetrahedra models has especially large errors for eccentric sources (Fig.2, see also [2,8]).

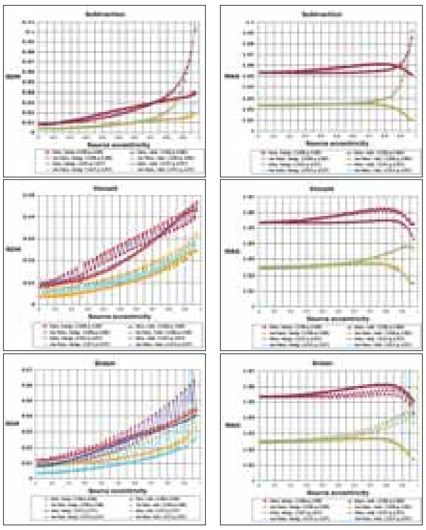

Figure 3

Four compartment sphere (92, 86, 80, 78mm; 0.33, 0.0042, 1.0, 0.33S/m) discretized with a 2mm hex and a 2mm ns-hex (426K nodes, 406K elements) mesh: RDM (left) and MAG (right) for the three FE forward modeling techniques subtraction (top), Venant (middle) and Green (bottom: highest RDM of 0.16 and MAG of 3.7 for ns-hex model is out of scale). Dipoles with fixed x and z and varying y-coordinate at realistic source eccentricities of 0 to 97% of the inner compartment at either (128,y,128) (along nodes and faces) or (127,y,127) (through element barycenters) were examined.

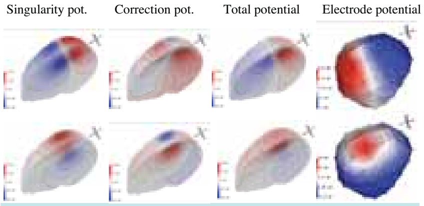

Figure 4

Subtraction approach in a realistically-shaped three compartment (skin, skull, brain; 0.33, 0.0042 and 0.33S/m) 2mm ns-hex (386K nodes, 366K elements) volume conductor: Results for the analytically computed singularity potential (left), the FE-based correction potential (left middle) and the resulting total potential in the volume conductor (right middle) and at the electrodes (right) for a tangentially (top) and radially oriented dipole (bottom).

Conclusions

Inverse source analysis: All presented numeric approaches can exploit the computationally efficient EEG/MEG lead field bases concept which reduces the "number of FE equation systems to solve" to the "number of sensors" [6,11,14]. Each FE based forward solution is then especially cheap for the direct potential methods [14]. With our current implementation, we recommend the choice of the Venant direct potential approach at least for those inverse methods exploiting influence matrices (beamformer, current density approaches, scanning methods). The direct potential approaches are less appropriate for inverse optimization methods in continuous parameter space (e.g., dipole fits using simplex optimization), because of the presented error curve oscillations, there might be a higher risk to get stuck in local minima.

Anisotropy: All three FE approaches can treat *remote tissue anisotropy* (skull, white matter) [15], but a clear theory for *local anisotropy* (grey matter) only exists for the subtraction approach [16]. Hex node-shifting should be used for the skin and skull compartments while it cannot be recommended to deform elements at the grey and white matter surfaces.

Perspective: The subtraction method is theoretically best understood and bears the highest future potential because higher FE Ansatz-functions and/or local refinement techniques should solve the accuracy problems when approaching a conductivity jump.

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