waLBerla: Brownian fluctuations in lattice Boltzmann simulations on mesoscales
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- waLBerla framework
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  - Parallel LB framework with adjustable functionality for various CFD applications
  - Adjustable functionality:
    - Flexible integration of new functionality
    - Specialization of functionality for efficiency improvements
  - CFD applications:
    - Homogeneous mixtures
    - Free-Surfaces
    - Brownian motion
    - Turbulence
    - Fluid structure interaction

- The fluctuating lattice Boltzmann method (FLB)
  - Lattice Boltzmann update rule
    \[
    f_i'(x,t) = f_i^* + \Delta (f_i^{neq})
    \]
  - Collision operator \( \Delta (f_i^{neq}) \) for FLB ([1]):
    \[
    \begin{align*}
    \gamma_i &= \sqrt{\mu(T) \rho \cdot a^T} \cdot f_i^{neq} \quad \text{Thermal normalisation} \\
    m_i &= \sum E_{ik} \gamma_i \quad \text{Mapping to momentum space} \\
    \gamma_i &= \gamma_i m_i + \phi_i t_i \quad \text{Collision update including Gaussian noise (}\phi_i t_i\text{)} \\
    m_i' &= \sum E_{ik} m_i' \quad \text{Mapping to velocity space} \\
    f_i^{neq} &= \sqrt{\mu(T) \rho \cdot a^T} \cdot \gamma_i \
    \end{align*}
    \]
  - Variances \( \varphi \) of Gaussian noise chosen such that
    \[
    \text{Var}(\varphi) \to \text{Theory-Enskog-Chapman} \quad \text{with}
    \]
    \[
    \text{Cov}(Q_{\varphi}(x,t), Q_{\varphi}(x',t')) = \frac{2k_B T}{\Delta x^T} \left[ \eta_{\text{bulk}} - 2 \frac{1}{3} \eta_{\text{bulk}} \right] \delta_{x'} \delta_{y'} + \eta_{\text{bulk}} \delta_{x'} \delta_{y'}
    \]

- Brownian fluctuations in fluid flow
  - FIG. 1: Example of Brownian fluctuations in fluid flow

- Mean squared particle displacements
  - FIGS. 4, 5: Numerical measurement of mean squared particle displacements

- references: