Distributed Asynchronous Jacobi Methods
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Motivation
- Solving $Ax = b$ is a common task in parallel computing
- Synchronous algorithms are dominated by:
  - individual slow processes
  - slow network connections
- Asynchronous algorithms avoid synchronization

Iterative Methods
- Synchronous iterative method using $p$ processes in parallel:
  $$x_i^{(j)} = f_i\left(x_1^{(j-1)}, x_2^{(j-1)}, \ldots, x_n^{(j-1)}\right) \quad \text{for} \quad i = 1, 2, \ldots, p$$
  - in iteration $j$, processes use other processes' data from the preceding iteration $j-1$
- Asynchronous iterative method using $p$ processes in parallel [1]:
  $$x_i^{(j)} = f_i\left(x_1^{(s_i(j))}, x_2^{(s_i(j))}, \ldots, x_n^{(s_i(j))}\right) \quad \text{for} \quad i = 1, 2, \ldots, p$$
  - in iteration $j$, processes use other processes' data from any iteration $s_i(j)$
- Asynchronous convergence detection [2]:
  - master process checks processes' individual convergence
  - individual convergence is reached if:
    - local residual < threshold
    - no iterations have been performed since the last data sending
    - all send operations are completed

Implementation
- Test problem: Laplace's equation $\Delta u(x, y) = 0$
- discretized with the finite difference method
- solved with the Jacobi method
- C++11 using Message Passing Interface (MPI) 3.0
- Algorithm
  ```
  while not converged globally {
    Jacobi iteration
    asynchronous communication
  }
  ```
- Asynchronous convergence detection [2]:
  - master process checks processes' individual convergence
  - individual convergence is reached if:
    - local residual < threshold
    - no iterations have been performed since the last data sending
    - all send operations are completed

Evaluation
- 400x400 grid point domain, homogeneously tile-partitioned
- 560 compute nodes with 2 Intel Xeon E5-2660 v2 processors
- 10 physical cores (+SMT) per processor = 40 processes per node
- Infiniband network with 40 GBit/s bandwidth

Cloud infrastructure with 19 virtual compute nodes based on:
- compute nodes with 2 Intel Xeon X5660 processors
- 6 physical cores (+SMT) per processor = 24 processes per node
- Ethernet network with 1 GBit/s bandwidth

References