New Finite Elements for Large-Scale Simulation of Optical Waves

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Outline

1. Problem
2. Trigonometric Finite Wave Elements
3. Numerical Results
4. Summary and Outlook
Distributed Feedback Laser

**Goal**

Simulation of the optical wave in DFB lasers

**DFB laser**

- Layers with different refraction indices (gratings) → Internal reflections of the optical wave
- Long resonator with length $L$
- Small stripe width of size $s$ of injection current $j$
Difficulties in the simulation of the optical wave

Goal
Simulation of the optical wave in DFB lasers

Difficulties
- Large-scale simulations of the wave equation require a large number of grid points
- Internal reflections \(\rightarrow\) Propagation in \(x\) and \(-x\) direction has to be treated simultaneously
State of the Art

Well-known methods for the simulation of optical waves

- Beam Propagation Method
- Finite Element Ray Method
- Partition of Unity Finite Element Method
- Finite Difference Time Domain Method
- Finite Element Method with standard Finite Elements

But: Internal reflections and large resonators cannot be simulated by these methods

→ Transfer Matrix Method (TMM)

But: Tapered lasers need a 2D-simulation
We are looking for Finite Elements that solve the wave equation as exact as the TMM → **Trigonometric Finite Wave Elements** (TFWE) as a linear combination of sine and cosine functions that approximate the behavior of an oscillating and internal reflected wave

Construct TFWE in 2D by a tensor product of 1D TFWE in propagation direction and linear nodal basis functions in perpendicular direction → Non-conforming TFWE in 2D

→ TFWE method combines advantages of TMM and FEM
Trigonometric Finite Wave Elements in 1D

1D linear nodal basis functions are multiplied by appropriate sine and cosine functions → Every node has three basis functions:

Remark: Multiplying the linear nodal basis functions with $\exp(\pm ikx)$ leads to numerical problems, as the basis functions are less orthogonal

$k$: wave number
Non-conformity of the 2D-TFWE

Wave number $k$ varies in y-direction $\rightarrow$ Discontinuity of the basis functions in y-direction from cell to cell
Oscillation Assumption

Let $u \in H^2(\Omega)$ oscillate with a frequency $\omega \approx ck$. This means, that $u = u^+ \exp(ikx) + u^- \exp(-ikx)$, where $u^+ \exp(ikx) \in C(\Omega)$, $u^- \exp(-ikx) \in C(\Omega)$, $|u^+|_{H^2(\tilde{\Omega}_h)} \ll |u|_{H^2(\Omega)}$, and $|u^-|_{H^2(\tilde{\Omega}_h)} \ll |u|_{H^2(\Omega)}$.
Approximation property in 1D

Oscillation Assumption

Let \( u \in H^2(\Omega) \) oscillate with a frequency \( \omega \approx ck \).

Theorem

Let \( u \in H^2(\Omega) \) satisfy the Oscillation Assumption. Then, there exists a constant \( C \) independent of \( h \) and \( k_{\text{max}} := \max_{1 \leq j \leq N} |k_j| \) such that

\[
|u - l_{h}^{\text{osc}}(u)|_{H^1(\Omega)} \leq C(k_{\text{max}} h + 1) h \left( |u^+|_{H^2(\tilde{\Omega}_h)} + |u^-|_{H^2(\tilde{\Omega}_h)} \right).
\]

\( \tilde{\Omega}_h \): \( \Omega \) without grid points  
\( l_{h}^{\text{osc}} : H^2(\Omega) \rightarrow V_h \): interpolation operator  
\( V_h \): TFWE space  
\( h \): mesh width  
\( k_j \): discretized wave numbers  
\( c \): velocity of light
Convergence of the 2D non-conforming TFWE

Theorem

Let \( u \in H^2(\Omega) \). Then, there exists a constant \( C \) independent of \( h := \max\{h_x, h_y\} \) such that

\[
\| u - u_h \|_{H^1(\tilde{\Omega}_h)} \leq C h \| u \|_{H^2(\Omega)}.
\]

Proof

- Lemma of Strang
- Detailed analysis of the non-conforming TFWE with the help of a computer algebra system

\( \tilde{\Omega}_h \): \( \Omega \) without horizontal grid lines

\( h := (h_x, h_y) \): tuple of mesh widths

\( u_h \): exact solution of the discretized problem
System of Coupled Partial Differential Equations

- Behavior of wave is described by

\[ 2i \frac{k(n_A)^2}{\omega} \frac{\partial E}{\partial t} = \triangle E + k(n_A)^2 E \]

- Equations for carrier densities \( n_A \) and \( n_B \)

\[
\begin{align*}
\frac{\partial n_A}{\partial t} &= \nabla(D_A \nabla n_A) + \frac{n_B}{\tau_{cap}} - \frac{n_A}{\tau_{esc}} - r_{rec,A}(|E|^2, n_A) \\
\frac{\partial n_B}{\partial t} &= \nabla(D_B \nabla n_B) + \eta_{i,leck} \frac{j}{qd_B} - \frac{n_B}{\tau_{cap}} \frac{d_A}{d_B} + \frac{n_A}{\tau_{esc}} \frac{d_A}{d_B} - r_{rec,B}
\end{align*}
\]

\( D_A, D_B \): ambipolar diffusion constants  \( \tau_{cap} \): effective carrier aligning time
\( r_{rec,A}, r_{rec,B} \): recombination densities  \( \tau_{esc} \): carrier emission time
\( d_A, d_B \): thickness of active zone, barriers  \( j \): current density
\( \eta_{i,leck} \): internal efficiency  \( q \): elementary charge
Spatial Hole Burning

Photon density $n$

Carrier density $n_A$

Spatial hole burning in areas of high photon density with period $\frac{\lambda}{2}$
Higher Order Modes

Photon density $n$ at stripe width $25\mu m$

Photon density $n$ at stripe width $40\mu m$
Tapering

Current density $j$

Photon density $n$

Tapered lasers combine the beam quality of a Ridge Waveguide laser with the high power known from Broad Area lasers.
Summary

- TFWE lead to better performance than standard FE
- Internal reflections can be simulated
- 2D-simulation of large resonators is possible

→ TFWE method combines advantages of FEM and TMM

- Approach shows laser-typical effects such as spatial hole burning
- Influence of stripe width and current to the order of the resulting mode can be examined

→ Simulation can support the tuning of DFB lasers
Outlook

- Extension of the simulation to 3D
- Introduction of multigrid method for solving the wave equation
- Fourier analysis of the optical wave in time
- Extension of the simulation to Maxwell’s equations for analyzing the polarization of the optical wave
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