Nonlinear Diffusion vs. Wavelet Based Noise Reduction in CT Using Correlation Analysis

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Motivation and Idea

- Radiation dose for CT scan between 25–500 times higher than for a single X-ray image

- Noise reduction in CT images provides a possibility to increase signal-to-noise ratio, hence giving more space for a further reduction of radiation dose

- Idea:
  - Estimate real image structure out of the correlations of two input data sets affected with uncorrelated noise and adapt to the spatially changing behavior of noise on CT images by estimating the local noise variance out of the difference of the input images
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- Keep medical relevant information!
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**Idea:** Estimate real image structure out of the correlations of two input data sets affected with uncorrelated noise and adapt to the spatially changing behavior of noise on CT images by estimating the local noise variance out of the difference of the input images
CT geometry

(a) CT geometry
Reconstruction

- A CT scanner acquires projection data
Reconstruction

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- The original object is reconstructed from them by filtered back projection (FBP). Mathematically an inverse Radon transform is required
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The original object is reconstructed from them by filtered back projection (FBP). Mathematically an inverse Radon transform is required.

The resulting image exhibits different kinds of (unknown) noise e.g. due to low radiation dose or the (approximate) reconstruction algorithm.
Reconstructed CT image

(b) Noisy CT image
Related Literature

A. Borsdorf, R. Raupach, and J. Hornegger.
Wavelet based Noise Reduction by Identification of Correlation.

A. Borsdorf, R. Raupach, and J. Hornegger.
Separate CT-Reconstruction for Orientation and Position Adaptive Wavelet Denoising.

P. Perona and J. Malik.
Scale-space and edge detection using anisotropic diffusion.
Denoising image by standard nonlinear diffusion
Denoising model

- We split the set of projections $P$ into even and odd numbered projections $P_1$ and $P_2$
- We reconstruct two CT images $P_1$ and $P_2$ containing the same object
- We denoise both CT images by nonlinear diffusion filtering or wavelets as done in [BRH06, BRH07]
**Variational model**

Visualization of average image

(e) Original \( \frac{u_1^0 + u_2^0}{2} \)

(f) Difference \( \frac{u_1^0 - u_2^0}{2} \)

**Figure:** One slice of the original average image of an abdomen CT scan and the difference image of size 512 × 512.
Variational model: Nonlinear diffusion

Energy minimization requires to solve the PDE [PM90]

\begin{align}
    u - u^0 &= \alpha \text{div}(\tilde{g}_{\sigma \eta} \nabla u) \quad (1a) \\
    u_1 - u_1^0 &= \alpha \text{div}(\tilde{g}_{\sigma \eta} \nabla u_1) \quad (1b) \\
    u_2 - u_2^0 &= \alpha \text{div}(\tilde{g}_{\sigma \eta} \nabla u_2) \quad (1c)
\end{align}

with Neumann boundary conditions. \( u \) is the denoised image and \( u_1, u_2 \) are required to compute the nonlinear diffusivity function \( \tilde{g}_{\sigma \eta} \), only.
Estimation of local correlation

Because of the spatially varying noise properties in CT images we consider a local neighborhood $\Gamma_x \subset \Omega$ around a pixel $x$. A local estimate for the correlation of two image regions can be computed by the \textit{weighted correlation coefficient}

$$c_w(x) = \frac{\text{Cov}_{w,u_1,u_2}(x)}{\sqrt{\text{Var}_{w,u_1}(x)}\text{Var}_{w,u_2}(x)}.$$  \hspace{1cm} (2)

Since in our case only the similarity between image regions is interesting, we set values of $c_w$ that are below 0 – and thus denoting anti-correlation – to 0. This yields a local similarity measure

$$C_w(x) = \begin{cases} c_w(x) & c_w(x) > 0 \\ 0 & c_w(x) \leq 0 \end{cases}.$$  \hspace{1cm} (3)
Local noise variance

Two input images provide the possibility to estimate the local noise variance $\text{Var}_\eta$ of the average of the images $u_1$ and $u_2$ by

\[
\sigma^2_\eta(x) := \beta \text{Var}_\eta(x) = \frac{\sum_{\tilde{x} \in \Gamma_x} w(\tilde{x}, x)(u_1(\tilde{x}) - u_2(\tilde{x}))^2}{4 \sum_{\tilde{x} \in \Gamma_x} w(\tilde{x}, x)}
\]

(4)

with scaling parameter $\beta \in \mathbb{R}^+$. 
Visualization of correlation and variance

(a) Correlation $C_w$

(b) Variance $\text{Var}_\eta$

**Figure**: Plots of corresponding local correlation $C_w$ and local variance estimate $\text{Var}_\eta$. 
We define the diffusivity function

\[
\tilde{g}_{\sigma, \eta}(x) := \begin{cases} 
  g_{\sigma, \eta} \left( \sqrt{\|\nabla u_1(x)\| \cdot \|\nabla u_2(x)\|} \cdot W(x) \right) & \text{if } W(x) > 0, \\
  g_{\sigma, \eta}(0) & \text{else}
\end{cases}
\]

based on the Tukey edge-stopping function

\[
g_{\sigma, \eta}(x) = \begin{cases} 
  \left( 1 - \left( \frac{x}{\sigma, \eta} \right)^2 \right)^2 & : |x| \leq \sigma, \\
  0 & : |x| > \sigma
\end{cases}
\]

with

\[
W(x) = 1 + \gamma(2C_w(x) - 1), \gamma \in \mathbb{R}.
\]

This damps high gradients in image regions with small similarity, e.g., in homogeneous regions, and enlarges the gradient where similarity is high, i.e., when image structure is present.
Experimental results

Multigrid solver

- nonlinearity is treated by inexact lagged diffusivity and a nonlinear FAS method
- cell-centered grid and standard transfer operators
- RBGS smoother
- C++ Code
Quantitative measurements on phantom data

(a) 10 HU  
(b) 60 HU  
(c) 100 HU

Figure: Simulated phantom CT scans with different contrast-to-noise levels.
Experimental results

Modulation transfer function I

Figure: MTFs achieved by denoising of phantoms for different contrast-to-noise levels at the edge. A pattern of a certain number of lp/cm is damped by the factor on the ordinate.
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Experimental results

Original noisy CT image

(a) Original
Denoising by presented approach

(b) 2D

(c) 3D
Experimental results

Difference images for presented approach

(d) 2D – original

(e) 3D – original
Figure: Comparison of $\rho_{50}$-values achieved for PDE or wavelet based method plotted against the contrast of the edge.
Comparison to wavelets II

(a) Wavelets 2D – original

(b) Wavelets 3D – original
Comparison to wavelets III

(c) Wavelets 2D – original
(d) Wavelets 3D – original
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Future work

- Speed up code: currently we denoise two $512^2$ images in 3 s and two $512^2 \times 16$ images in 90 s
- Evaluate use of anisotropic diffusion
- Compare to patch based denoising approaches