Simulations of Particle-laden Flows with the Lattice Boltzmann Method

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Outline

- The lattice Boltzmann method (LBM)
- Particle coupling algorithms
- Momentum exchange method
- Benchmark: single moving sphere
- Conclusion
The lattice Boltzmann method (LBM)
Classical CFD

velocities ($\vec{u}$) and pressure ($p$)

LBM

originates from statistical mechanics

- distribution functions ($f_i$), corresponding to lattice velocities $\vec{c}_i$

D2Q9 model

density: $\rho = \sum_i f_i$
momentum: $\rho \vec{u} = \sum_i f_i \vec{c}_i$
LBM equation

\[ f_i(\tilde{x} + \tilde{c}_i, t + 1) = f_i(\tilde{x}, t) + \frac{1}{\tau} \left( f_{i}^{eq}(\rho, \tilde{u}) - f_i(\tilde{x}, t) \right) \]

- Meaning: relax the \( f_i \) linearly towards their equilibrium values
- Relaxation parameter \( \tau \) determines the viscosity
- Approximates the Navier-Stokes equations
- Algorithm:

  In each timestep \( t \):
  1. Collide: \( \tilde{f}_i(\tilde{x}, t) = f_i(\tilde{x}, t) + \frac{1}{\tau} \left( f_{i}^{eq}(\rho, \tilde{u}) - f_i(\tilde{x}, t) \right) \)
  2. Stream: \( f_i(\tilde{x} + \tilde{c}_i, t + 1) = \tilde{f}_i(\tilde{x}, t) \)

- Alternatives for improved stability and accuracy:
  - TRT (two relaxation times)
  - MRT (multiple relaxation times)
Collide step

1. Collide: \( \tilde{f}_i(\vec{x}, t) = f_i(\vec{x}, t) + \frac{1}{\tau} \left( f_i^{eq}(\rho, \vec{u}) - f_i(\vec{x}, t) \right) \)
Stream step

2. Stream: $f_i(\tilde{x} + \tilde{c}_i, t + 1) = \tilde{f}_i(\tilde{x}, t)$
Particle coupling algorithms
Overview

- Immersed boundary method [1]
  - Use Lagrangian particles to track the particle’s surface
  - Coupling via body force in cells at particle’s boundary

- Noble-Torczynski method [2]
  - Use information about solid volume fraction of each cell
  - Coupling via special collision term for cells inside particle

- Momentum exchange method [3]
  - Explicitly map body into domain
  - Coupling via boundary conditions

[1]: Peskin – Numerical analysis of blood flow in the heart, 1977
[2]: Noble, Torczynski – A lattice-Boltzmann method for partially saturated computational cells, 1998
[3]: Ladd – Numerical simulation of particulate suspensions via a discretized Boltzmann equation[…], 1994
Momentum exchange method: overview

No-slip boundary condition

Fluid flow simulation on discretized domain (LBM)

[www.walberla.net]

Rigid body simulation (e.g. Discrete Element Method)

Momentum exchange
Particle mapping

- Mark cells inside particle as solid
**Particle → Fluid coupling**

- No-slip boundary condition:

  \[ f_i(\vec{x}, t + 1) = f_i(\vec{x}, t) - 6w_i \rho \vec{u}_p \cdot \vec{c}_i \]

- Reflects distribution function at boundary and adds contribution depending on the particles’ velocity \( \vec{u}_p \)

Alternatives that use exact boundary location information:

- CLI [1]
- Multifreflection [1]
- Etc.

[1]: Ginzburg et al. – Two-relaxation-time lattice Boltzmann scheme: About parameterization, velocity, pressure and mixed boundary conditions, 2008
Fluid → Particle coupling

- Force based on momentum exchange (ME) [1]
  \[
  \vec{F}_i(\vec{x}_b) = f_i(\vec{x}, t) \left[\vec{c}_i - \vec{u}_p\right] - f_i(\vec{x}, t + 1) \left[\vec{c}_i - \vec{u}_p\right]
  \]
- Obtain total force and torque on particle by summing up all local contributions

[1]: Wen et al. – Galilean invariant fluid-solid interfacial dynamics simulations in lattice Boltzmann simulations, 2014
Speciality: Refilling of uncovered cells

Variants of restoration of distribution functions:

• Set to equilibrium $f_i^{eq}(\rho, \vec{u})$ based on average density of surrounding fluid cells

• Extrapolation from particle normal direction

  ➢ Constraint: Velocity has to match particle velocity in this cell
Overall coupling algorithm

For each timestep $t$:
1. Map particles into fluid domain
2. Restore missing information in uncovered cells
3. Apply no-slip boundary conditions at particles
4. Calculate forces on particles
5. Carry out LBM step
6. Carry out rigid body solver step (collision detection, time integration)
Benchmark: single moving sphere
Setup

- Single heavy sphere is placed inside a horizontally periodic channel
- Constant inflow from bottom plane
- Choose $\vec{F}_g \approx \vec{F}_{Drag}$
- Important parameter: Galileo number $G = \sqrt{|\rho_p| - 1 |g| D^3}$
  - characterizes different flow regimes

- Compare with finite volume (FV) and accurate spectral element results [1]

[1]: Uhlmann, Dušek – The motion of a single heavy sphere in ambient fluid[...], 2014
Galileo = 178: Steady oblique

- Compare relative vertical particle velocity $u_{pV}$, recirculation length $L_r$, velocity profiles, etc.
- Additionally: Horizontal $u_{pH}$ and rotational $\omega_{pH}$ particle velocity

<table>
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<tr>
<th>Method</th>
<th>$u_{pH}$</th>
<th>$E(u_{pH})$</th>
<th>$\omega_{pH}$</th>
<th>$E(\omega_{pH})$</th>
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</thead>
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<tr>
<td>Spec.Elem.</td>
<td>0.1245</td>
<td>-</td>
<td>0.0137</td>
<td>-</td>
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<tr>
<td>FV (D=48)</td>
<td>0.1028</td>
<td>1.60%</td>
<td>0.0089</td>
<td>0.35%</td>
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<td>LBM (D=24)</td>
<td>0.0891</td>
<td>2.61%</td>
<td>0.0245</td>
<td>0.80%</td>
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<tr>
<td>LBM (D=36)</td>
<td>0.0975</td>
<td>1.99%</td>
<td>0.0136</td>
<td>0.00%</td>
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<tr>
<td>LBM (D=48)</td>
<td>0.1002</td>
<td>1.79%</td>
<td>0.0091</td>
<td>0.34%</td>
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</tbody>
</table>
Galileo = 190: Oscillating oblique

- Oscillating motion with a fixed frequency $f$
- Comparison of average and fluctuation quantities

<table>
<thead>
<tr>
<th>Method</th>
<th>$f$</th>
<th>$E(f)$</th>
</tr>
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<tbody>
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<td>Spec.Elem.</td>
<td>0.071</td>
<td>-</td>
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<tr>
<td>FV (D=48)</td>
<td>0.0683</td>
<td>3.80%</td>
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<td>LBM (D=36)</td>
<td>0.0863</td>
<td>21.51%</td>
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<td>LBM (D=48)</td>
<td>0.0654</td>
<td>7.85%</td>
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</table>
Conclusion
Conclusion

- LBM with momentum exchange method is a viable choice for simulations of particle-laden flows
- No restrictions regarding shape of particles
- Well-suited for large computations on clusters due to local nature of LBM [1]
- Other LBM coupling algorithms exist but a rigorous direct comparison is missing
  - Use benchmark of single moving sphere for comparing different methods and algorithms

[1]: Godenschwager et al. – A Framework for Hybrid Parallel Flow Simulations with a Trillion Cells in Complex Geometries
Thank you for your attention!

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